# What Is the Cost of Financial Flexibility? Theory and Evidence for Make-Whole Call Provisions

**Eric Powers and Sergey Tsyplakov***

*Firms commonly incorporate make-whole call provisions in their newly issued debt, presumably to improve their ability to retire debt early if circumstances require. In return for increased financial flexibility, firms must compensate bondholders with additional (incremental) yield. To estimate theoretical incremental yields, we use and calibrate a structural model for a large sample of callable and noncallable US corporate bonds issued between 1995 and 2004. In a frictionless model where calls occur only when they are in-the-money, theoretical incremental yields average approximately 2 basis points (bp). In an extended model that incorporates taxes, transactions costs, and randomly occurring exogenous events requiring early bond retirement, incremental yields average approximately 5 bp. Empirical analysis, however, indicates that observed incremental yields are significantly greater than model-generated values, averaging between 13 and 24 bp. In the later years of our sample period, however, observed incremental yields begin to converge to model-generated values.*

By definition, "financial flexibility" is irrelevant in a frictionless capital market. In the real economy with frictions, however, it is well documented that corporate executives place significant value on having financial flexibility (Graham and Harvey, 2001). Unfortunately, nothing of value comes for free. For example, if maintaining flexibility requires keeping leverage low so that additional debt can be issued in the future, valuable tax shields are foregone (Graham, 2000). Alternatively, if maintaining flexibility requires incorporating features like call provisions in newly issued debt to be able to retire it early (Smith and Warner, 1979; Narayanan and Lim, 1989), bondholders must be compensated with higher yields. Whatever action the firm is contemplating to improve financial flexibility, understanding the costs involved is critical.

Our focus is on understanding the costs associated with make-whole call provisions—a financial innovation that is particularly useful for firms that want the flexibility to retire debt early. With a make-whole call provision, the call price is calculated as the maximum of par value or the present value of the bond's remaining payments. The discount rate used in the calculation is the prevailing comparable maturity Treasury yield plus a contractually specified spread called the make-whole premium. Typically, the make-whole premium ranges between 0 and 50 basis points (bp). The innovative characteristic of a make-whole call provision is that the call price floats inversely with risk-free rates.

*We thank Brent Ambrose, David P. Brown, Shingo Goto, Brad Jordan, Ken Kavajecz, Ajay Khorana, Steve Mann, Sattar Mansi, Ted Moore, Greg Niehaus, Mark Ready, Ellen Roueche, Martin Ruckes, Stathis Tompaidis, Toni Whited, an anonymous referee, and seminar participants at Georgia Institute of Technology, Notre Dame University, the University of South Carolina, the University of Wisconsin at Madison, the 2004 Financial Management Association annual meeting, and the 2005 Southern Finance Association annual meeting for helpful comments.*

*Eric Powers and Sergey Tsyplakov are Associate Professors of Finance at the Moore School of Business, University of South Carolina in Columbia, SC 29208.*

Without a call provision, firms typically must rely on bond tender offers to retire debt early. Successful tender offers, however, can potentially be quite costly.[1] With a call provision, a firm can retire debt early via either the call provision or a tender offer—whichever is cheaper. Thus, a make-whole call provision improves a firm's financial flexibility by capping the price of a successful bond tender offer.

While traditional fixed-price call provisions also cap the price of a successful tender offer, make-whole call provisions are a superior mechanism for improving financial flexibility for several reasons. First, a make-whole call provision's floating call price eliminates interest rate risk.[2] Thus, compared to a fixed-price call provision, up-front costs for a make-whole call provision should be lower. Second, to moderate the interest rate risk that bondholders are exposed to, most fixed-price call provisions incorporate several years of call or refund protection, thus limiting the financial flexibility provided by the call provision. In contrast, make-whole call provisions almost never incorporate a call protection period. Finally, fixed-price call prices are likely to be greater than successful tender offer prices if interest rates have risen since the bond was issued. The floating call price of a make-whole call provision negates this limitation.

Given the beneficial characteristics of make-whole call provisions, it is not surprising that there has been an explosion in their popularity since the mid-1990s.[3] In 2005, for example, make-whole call bonds accounted for more than 50% of the volume of newly issued corporate bonds in the United States (see Figure 1). The willingness of firms to incorporate make-whole call provisions in new debt issues indicates that the perceived benefits of increased financial flexibility more than make up for the call provision's up-front cost. While it is difficult to quantify the ex post flexibility benefits of a make-whole call provision, quantifying the ex ante cost is feasible.

To assess the cost of a make-whole call provision, we develop a structural credit spread model with stochastic interest rates, stochastic firm value, and endogenous default decisions. We begin with the standard structure of a frictionless economy where call executions are endogenous, only occurring when call provisions are in-the-money. Since the primary benefits of financial flexibility occur when the frictionless assumption is relaxed, we then extend our model by incorporating frictions that increase the value of the call provision. The primary friction is a randomly arriving exogenous event process that compels the firm to retire debt early for reasons other than the call being in-the-money. We also include capital gains taxes and reinvestment transactions costs incurred by debtholders subsequent to early retirement. Thus, in the extended model, bondholders must receive additional ex ante compensation due to capital gains taxes owed, and transaction costs incurred, when calls occur. Additionally, bondholders must be compensated for the fact that a make-whole call provision acts as a cap on the price of a successful tender offer.

In both the frictionless and the more realistic environment, we parameterize our model using contemporaneous risk-free yield curve information as well as firm- and bond-specific

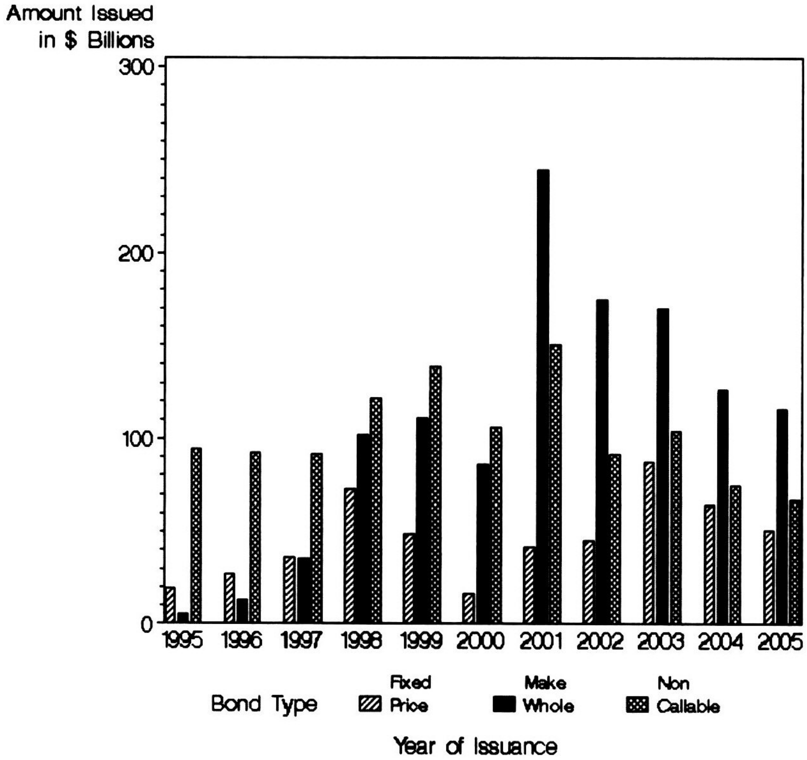
[1] On September 4, 1997, General Electric Corporation tendered for the 10.5% Greenwich Air Services note maturing June 2006 (GE had previously acquired Greenwich). GE offered $1185.71 per bond—a price that equates to a 0 bp spread above risk-free rates. At that time, similar bonds were trading at spreads of more than 200 bp. Thus, the tender price was almost $60 greater per bond than the market price. Bondholders tendered 92% of outstanding bonds.

[2] As noted by Güntay, Prabhala, and Unal (2004), there has been a significant decline in the prevalence of fixed-price call provisions that coincides with an increase in the availability of over-the-counter interest rate derivatives. Thus, firms are now more capable of managing interest rate risk without attaching fixed-price call provisions to their newly issued debt.

[3] The first publicly issued US corporate bond that we can identify with a make-whole call provision is the 8 1/8%, 15-year note issued by Harvard University on April 21, 1992. Kaplan (1998) and Jacoby and Stangeland (2004) state that make-whole call provisions appeared in Canadian corporate bonds as early as 1987. As noted by Kahan and Tuckman (1993) and Kwan and Carleton (2004), make-whole call provisions were prevalent in privately issued debt well before they were prevalent in publicly issued debt in either country.

**Figure 1. Annual Dollar Volume**

The columns display the nominal dollar amount of noncallable, fixed-price callable, and make-whole callable corporate debt issued in the United States per year. Included issues have the following characteristics: maturity of at least one year, denominated in US dollars, offering amount greater than $25 million, fixed semi-annual coupon, not asset-backed, not putable, without a sinking fund, not a Yankee bond, not a medium-term note, not part of a unit offering, and listed as a corporate debenture. There were a total of three make-whole bonds issued in 1992, four issued in 1993, and two issued in 1994. These years are not displayed.




characteristics at origination for a sample of over 3,000 noncallable and make-whole call bonds issued between 1995 and 2004. In a frictionless economy with strictly endogenous call decisions, the model-generated incremental yield for our sample of make-whole call bonds averages 2.1 bp. While 2.1 bp might seem exceedingly small, it is understandable when one considers that in a frictionless environment, calls are strictly endogenous and only occur when the credit spread for an equivalent noncallable bond narrows to less than the make-whole premium. Since make-whole premiums for investment grade bonds are typically set at only 15% of the prevailing credit spread at issuance (Powers and Sarkar, 2006), the likelihood of credit spreads narrowing by this much is low. Even then, the maximum value that equityholders can expropriate from bondholders is limited. Hence, the required up-front incremental yield is low. In the more realistic environment incorporating the exogenous requirement to retire bonds early as well as taxes and transactions costs, the model-generated incremental yield averages 5.4 bp. After controlling for a variety of other factors such as liquidity and the macroeconomic credit environment, however, regression analysis indicates that observed incremental yields range between 13 and 24 bp. Thus, observed incremental yields are significantly greater, on average, than the values predicted by our model.[4]

While observed incremental yields are significantly greater than model-generated incremental yields across the entire time spanned by our sample, there is evidence that the gap between observed and model-generated incremental yields is narrower for the most recent years. Indeed, for the last three years of our sample (2002-2004), observed incremental yields declined to less than half of what they were when make-whole call provisions were first becoming prevalent (1995-1998). Over time, investors seem to be realizing that make-whole call provisions do not contain the same degree of interest rate risk as fixed-price call provisions and are adjusting observed incremental yields accordingly.

The increased usage of make-whole calls indicates that borrowing firms are willing to offer additional yield to obtain the financial flexibility associated with a make-whole call provision. If the flexibility gained by the firm does not impose significant extra costs on bondholders, however, it is unclear why observed incremental yields have been as high as our analysis suggests. One potential explanation is that the decision to incorporate a make-whole call provision is endogenous and this endogeneity biases our estimated coefficients. Our results, however, are robust to regression specifications that specifically account for endogeneity. Indeed, the empirically observed incremental yields are greatest when we account for endogeneity. Thus, while the disparity between observed and model-generated incremental yields appears to be moderating, in the end we are left with an unresolved puzzle as to why incremental yields have been so high.

The remainder of the paper is as follows. In Section I, we develop the model. The data are described in Section II. In Section III, we use the data to calibrate the model for individual bonds and calculate model-generated credit spreads. The model-generated credit spreads are compared to observed at-issue credit spreads of both make-whole call and noncallable bonds in Section IV. In Section V, we account for possible endogeneity and present evidence on the evolution of incremental yields over time. Finally, a conclusion is provided in Section VI.

## I. Model

### A. Time Line

We begin with a basic structural model in an economy with no imperfections. At time zero the firm borrows an exogenous amount. The debt is a coupon bond with principal due at maturity and is either non-callable or has a make-whole call provision. The value of the firm's assets and the risk-free rate are assumed to be correlated stochastic processes with a constant correlation coefficient.[5]

At each instant, the firm decides to make the coupon payment, or to default, in which case bondholders recover the firm's assets net of default costs, and equityholders get nothing. For a callable bond, if default does not occur, the firm must decide whether to call the bond or leave it in place. If called, the call price is calculated as the maximum of par value or the present value

[4] An additional 10-20 bp in yield may seem negligible; however, it is substantial when compared to our sample's median credit spread at origination of 113 bp. In dollar terms, for a representative par bond with 15 years to maturity, a 7% coupon, and an offering amount of $500 million, an additional 10 bp in yield reduces offering proceeds by $4.57 million. This is almost 1% of gross proceeds and is a sum that should not be casually disregarded.

[5] Kim, Ramaswamy, and Sundaresan (1993) and Acharya and Carpenter (2002) use a similar setting for pricing coupon debt.

of the remaining scheduled debt payments, where the discount rate is the prevailing risk-free rate plus the make-whole premium.

The firm is assumed to make optimal default and call decisions. Default occurs when the value of the firm's assets declines and the market value of the firm's equity becomes zero. In contrast, a call occurs when the value of assets increases and the firm is able to issue an equivalent bond with a yield that is less than the risk-free rate plus the make-whole call premium of the existing bond. The value of the call provision is the present value of expected wealth transfers from bondholders to equityholders. Later, we modify the model by incorporating selected market imperfections that are expected to increase the incremental yield.

## B. Core Model Components

### 1. The Interest Rate Process

The short-term risk-free rate $r_t$ is assumed to follow a mean-reverting square root stochastic diffusion process, described by the one-factor Cox, Ingersoll, and Ross (1985) (CIR) model:

$$dr_t = \kappa_r(r^* - r_t)dt + \sigma_r\sqrt{r_t}dW_r, \tag{1}$$

where $\kappa_r$ is the mean-reversion rate, $r^*$ is the long-term level to which the short-term rate reverts, $\sigma_r$ is the instantaneous volatility for the short-term rate, and $W_r$ is a standard Wiener process under the risk-neutral measure.

### 2. Value of the Firm's Assets

We assume that the value of the firm's assets follows a log-normal stochastic process:

$$\frac{dV}{V} = (r - \alpha)dt + \sigma_V dW_V, \tag{2}$$

where $\alpha$ is the payout rate, $W_V$ is a Weiner process under the risk-neutral measure, and $\sigma_V$ is the instantaneous volatility coefficient. The Wiener processes $W_r$ and $W_V$ are correlated with correlation coefficient $p$.

### 3. Bond Types

We assume that the firm issues a bond that has a continuous coupon payment rate of $c$ per unit of time and a balloon payment $F$ at maturity $T$. We consider two types of bonds: 1) a bond with a make-whole call provision and 2) an option-free or noncallable bond. The call price at which the firm can call its debt is calculated as the present value of the remaining coupon payments $c$ and balloon payment $F$ discounted at the risk-free rate $r$ plus some prespecified premium $m > 0$. Additionally, there is a floor for the call price specifying that it cannot be lower than the par value of the bond $Par$.[6] For a bond that has $T - t$ remaining until maturity, the call price is given by

$$M(r, c, F, m, T - t) = \max\left\{Par, \int_t^T ce^{-m(x-t)}P(r, t, x)dx + F \cdot P(r, t, T)e^{-m(T-t)}\right\}, \tag{3}$$

[6]If the bond is traded at par at origination, then par value equals balloon payment: $Par = F$.

where $P(r, t, x)$ is the price at time $t$ of a risk-free zero-coupon bond paying \$1 at time $x$, calculated according to the CIR formula

$$P(r,t,x) = \left(\frac{2\gamma e^{(\kappa_r-\gamma)(x-t)/2}}{2\gamma + (\kappa_r - \gamma)(1 - e^{-\gamma(x-t)})}\right)^{2\kappa_r r^*/\sigma_r^2} e^{rB(x-t)}, \quad \text{where} \tag{4}$$

$$B(x-t) = \frac{-2(1 - e^{-\gamma(x-t)})}{2\gamma + (\kappa_r - \gamma)(1 - e^{-\gamma(x-t)})}, \quad \text{and} \quad \gamma = \sqrt{\kappa_r^2 + 2\sigma_r^2}. \tag{5}$$

The last two terms in Equation (3) are the present values of the coupon payments and of the balloon payment discounted at the risk-free rate plus the premium $m$.

## 4. Value of the Firm's Equity

For a levered firm having debt with characteristics $c$, $F$, $T$, $M$, where $M(r, c, F, m, T - t)$ is the call price for each $t < T$ and risk-free rate $r$, the value of the firm's equity is $E(V, r, t)$. Equity's value depends on the optimal default and call strategies and is given by

$$E(V,r,t) = \max\left\{0, \max_{\tau}\left[\mathrm{E}(V - M(r,c,F,m,T-\tau))e^{-\int_t^\tau r_y dy},\right.\right.$$
$$\left.\left.\mathrm{E}\left(\int_t^\tau (\alpha V(x) - c)e^{-\int_t^x r_y dy}dx + \delta(T-\tau)e^{-\int_t^T r_y dy}\max(0, V(T) - F)\right)\right]\right\}, \tag{6}$$

where E is the expectation under the risk-neutral measure, and $\tau$ is the stopping time that corresponds either to the time of default, or bond call, or maturity. The function $\delta$ in Equation (6) is given by $\delta(x) = 0$ if $x \neq 0$ and $\delta(0) = 1$.

Given the value of the short-term interest rate $r$, and the firm's assets $V$, the value of equity $E(V, r, t)$ can be determined by maximizing the expected value of equity, over all possible call and default strategies:

$$E(V(t), r(t), t) = \max\{0, \quad \max(V(t) - M(r,c,F,m,T-t), \quad [\alpha V - c]dt$$
$$+ e^{-r(t)dt}\mathrm{E}_Q[E(V(t+dt), r(t+dt), t+dt)])\}. \tag{7}$$

At the debt maturity date $T$, the value of the firm's equity is the greater of zero and the difference between the value of the firm's assets and the balloon payment,

$$E(V,r,T) = \max(V - F, 0). \tag{8}$$

Any time prior to maturity, $t < T$, the value of equity $E$ is given as the solution to the following PDE:

$$\frac{\sigma_r^2 r}{2}E_{rr} + \frac{\sigma_V V^2}{2}E_{VV} + \rho\sigma_V\sigma_r V\sqrt{r}E_{rV} + \kappa_r(r^* - r)E_r + (r - \alpha)E_V$$
$$+ E_t + \alpha V - c - rE = 0, \quad E \geq 0, \tag{9}$$

with additional free boundary conditions that characterize the boundaries where debt is called and where the firm defaults. The call boundary (if applicable) should satisfy

$$E(V, r) > V - M(r, c, F, m, T - t). \quad (10)$$

### 5. Valuation of the Firm's Debt

To calculate the value of the debt $D$, we need to consider the firm's default and call strategy (if applicable). At maturity $T$, the debt value is given by the following:

$$\begin{cases} D(V, r, T) = F, & \text{if } V \geq F \\ D(V, r, T) = (1 - DC) \cdot V, & \text{otherwise,} \end{cases} \quad (11)$$

where $DC$ represents proportional default costs ($0 \leq DC \leq 1$). Prior to maturity, the value of debt satisfies

$$\frac{\sigma_r^2 r}{2} D_{rr} + \frac{\sigma_V^2 V^2}{2} D_{VV} + \rho\sigma_V\sigma_r V\sqrt{r} D_{rV} + \kappa_r(r^* - r)D_r + (r - \alpha)D_V + Dt - rD + c = 0. \quad (12)$$

In the equation, the subscripts denote partial derivatives. There is a boundary condition for the value of debt when equity is valueless, the firm defaults, and assets are transferred to bondholders.

$$D(V, r, t) = (1 - DC) \cdot V, \quad \text{if} \quad E(V, r, t) = 0. \quad (13)$$

For a callable bond there is another free boundary condition that corresponds to the case where the firm buys the debt back at the make-whole call price $M(r, c, F, m, T - t)$:

$$D(V, r, t) = M(r, c, F, m, T - t) \quad \text{if} \quad E(V, r, t) = V - M(r, c, F, m, T - t). \quad (14)$$

Equations for the value of equity and debt can be solved numerically for both callable and noncallable bonds.

## C. Real-World Frictions

In a frictionless environment, incremental yield purely reflects the economic value of the call provision—it is compensation for expected wealth transfers from bondholders to shareholders when call provisions are optimally exercised. This expected wealth transfer clearly depends on the magnitude of the make-whole premium $m$. As noted in the introduction, make-whole premiums are usually quite low, being set at 15% of the prevailing credit spread at issue. Thus, actual calls will be unlikely and the economic value of the call provision in a frictionless environment will be minimal. This is confirmed when we subsequently parameterize our model.

As emphasized in the introduction, firms gain financial flexibility when a make-whole call provision is attached to a bond. Specifically, a make-whole call provision provides a mechanism for retiring an entire debt issue early. By definition, in a frictionless market, this flexibility is irrelevant to the firm. However, in the real world, being able to retire debt early at a predetermined spread above Treasuries can be valuable to the firm. To reflect the real-world environment in which financial flexibility is valuable, we extend our frictionless model by incorporating an exogenous event process that requires the firm to retire debt early. In addition, we also include the affects of capital gains taxes and transactions costs incurred by bondholders.[7]

### 1. Exogenous Early Bond Retirement

With a noncallable bond, if a situation arises where the firm must retire the bond early, the firm will normally attempt a bond tender offer. In a tender offer, the firm (via an information agent) contacts individual bondholders and offers to buy back their bonds at a specified price during a two- to six-week tender offer window. Bondholders can then opt to tender their bonds or abstain and remain bondholders. To motivate bondholders to tender, tender offer prices are usually 3% to 6% greater than the current market value of the bond (Mann and Powers, 2007).

Make-whole call provisions are beneficial to the firm in these situations because they function as a cap on the price of a 100% successful tender offer. Nothing prevents a firm with a make-whole call bond from attempting to tender for the bond at a price that is lower than the calculated make-whole call price.[8] However, if a tender offer appears too expensive, or is unlikely to succeed, the firm can still utilize the make-whole call provision—a firm with noncallable debt does not have this luxury. Since holders of make-whole call bonds will receive less on average than will holders of equivalent noncallable bonds when the firm retires debt early, make-whole call bondholders will demand additional ex ante compensation beyond the pure economic value of the call provision.

We modify the model by incorporating a Poisson process for an exogenous event that induces the firm to retire the bond early. We assume the arrival rate $\lambda$ is constant and independent of any model parameters. If such an event arrives, the firm with a noncallable bond tenders for its bond at the risk-free rate plus a tender spread $z\,(V, r, T-t)$ that depends on the credit quality of the bond subject to the tender. A firm with a make-whole call bond can also choose to call at the make-whole call price if it is advantageous to do so. Note that we still assume that the firm with a make-whole call bond can call its bond endogenously, that is, when the call is in-the-money.

### 2. Capital Gains Taxes

Whether the reason for executing a make-whole call is exogenous or endogenous, a taxable investor will owe capital gains taxes whenever the call price is greater than the investor's basis value. Since the make-whole call price has a floor at par, taxable investors who purchase a bond when issued will invariably owe capital gains taxes if the bond is called. To reflect this, we adjust the model such that if a call or tender offer is executed, bondholders must pay capital gains taxes $\phi$ on the difference between the call price and par value.

### 3. Transactions Costs

A significant percentage of corporate bonds are held by institutional investors who often have well-defined investment strategies. For example, the institution might follow a passive strategy

[7]In our extended model, we incorporate only imperfections that generate an increase in incremental yields. Alternative imperfections, however, could conceivably have the opposite effect. For example, firms might delay calls due to transactions costs incurred when calling (Mauer, 1993), or due to concerns about wealth transfers resulting from temporary capital structure changes (Longstaff and Tuckman, 1994), or simply because a suboptimal call policy is employed (King and Mauer, 2000). Anything that delays calls beyond the point predicted by our model will necessarily reduce model-generated incremental yields.

[8]In April 1999, RJR Nabisco tendered for two make-whole call bonds—an 8.25% note due 2004 (make-whole premium of 15 bp) and a 8.5% note due 2007 (make-whole premium of 25 bp). In the tender, Nabisco offered bondholders a fixed-spread above Treasuries of 95 bp for the 8.25% notes and 135 bp for the 8.5% notes. The percentage of bonds tendered was 98% for the 8.25% note and 96% for the 8.5% note.

designed to match the risk profile and return of a benchmark portfolio. Alternatively, the institution might follow an active strategy designed to capitalize on risk-arbitrage opportunities. In either case, exercise of a make-whole call provision or a tender offer can upset the investment strategy and force the institution to rebalance the portfolio at an inopportune time. We incorporate this "inconvenience," assuming that if the firm retires its debt, bondholders must pay transaction costs $\theta$ that are proportional to the balloon payment of the bond.[9] The details of valuing equity and debt after incorporating these three frictions are discussed in the Appendix.

## II. Data

We calibrate and test our model using a sample of bonds drawn from the Fixed Income Securities Database (FISD). The selected bonds have the following characteristics: 1) issued on or after January 1, 1995, 2) maturity of at least one year, 3) denominated in US dollars, 4) offering amount of at least $25 million, 5) fixed semi-annual coupon, 6) not asset-backed, 7) not putable, 8) without a sinking fund, 9) not a Yankee bond, 10) not a medium-term note, 11) not part of a unit offering, 12) not convertible, and 13) listed as a corporate debenture. These screens result in an initial sample of 2,798 noncallable bonds, 2,251 bonds with fixed-price call provisions, and 3,166 bonds with make-whole call provisions. Using the six-digit CUSIP of the issuer, we then match each noncallable and make-whole callable bond to the corresponding prior fiscal year record in Compustat, the source for most of the issuer-specific information required for our pricing model. Finally, we require at least one year's worth of daily Center for Research in Security Prices (CRSP) stock return data for the issuer for the year prior to the issuance date. Our final sample is composed of 1,506 noncallable bonds and 1,540 make-whole callable bonds for which we are able to identify all of the data items required to parameterize our model.

In Table I, Panel A, we present means and medians for characteristics of the noncallable and make-whole call bonds in our final sample.[10] The median make-whole call bond has a rating of Baa, 10 years to maturity, and an offering amount of $300 million. In contrast, the median noncallable bond has a rating of A–, 10 years to maturity, and an offering amount of $250 million. We also count the number of restrictive covenants present in each bond—the median make-whole call bond has four restrictive covenants while the median noncallable bond has three.

Following Eom, Helwege, and Huang (2004), we calculate credit spread as the bond's yield-to-maturity at issue minus the equivalent maturity Constant Maturity Treasury (CMT) yield. In situations where an equivalent maturity CMT yield is unavailable, we interpolate linearly using the two closest maturity CMT yields. The mean and median CMT spreads for the noncallable bonds are 113 bp points and 88 bp, respectively. Consistent with their lower ratings and prevalence later in the sample period when credit spreads are wider, the mean and median CMT spreads for the make-whole call bonds are statistically significantly greater at 166 bp and 144 bp, respectively. We

[9] In subsequent calculations, we impose an artificial restriction that no calls can occur within three months of the bond's maturity date. We do this because model-generated credit spreads decline to zero for all but the riskiest bonds as the maturity date approaches. This is a common outcome for structural models. If the bond has a make-whole call provision, then the likelihood of a call approaches one as long as the make-whole premium is greater than zero. This has minimal impact on the incremental yield of a make-whole call provision in a world devoid of imperfections. However, when we include transactions costs, the result is that these costs are imposed far too frequently unless something like the three-month barrier is applied.

[10] Characteristics of the final sample bonds do not differ materially from bonds that were excluded due to missing data.

## Table I. Bond Characteristics, Make-Whole Premiums, and Issuing Firm Characteristics

Panel A shows the bond characteristics: *Rating* is the ordinalized bond rating (AAA = 1, AA+ = 2, etc.), *Maturity* is years to maturity, *Offering Amount* is the total face value in millions of nominal dollars, *Total Restrictive Cov* is the sum of the number of restrictive covenants present in the bond's indenture, *CMT Spread* is the actual yield-to-maturity at issue minus the equivalent maturity Constant Maturity Treasury yield, *FMY Spread* means yield-to-maturity at issue minus the equivalent maturity Bloomberg Fair Market Yield. Columns 2 and 3 present results for the bonds where we were able to gather sufficient issuer-specific information to calibrate our model. Column 4 displays $p$-values for a $t$-test and a Wilcoxon rank-sum test comparing make-whole call bonds and noncallable bonds from columns 2 and 3. Panel B shows summary statistics for make-whole premium and credit spread by initial Moody's bond rating and bond maturity. The first line in each cell reports mean (median) make-whole premium. The second line in each cell reports mean (median) *CMT Spread*, measured as yield-to-maturity at issue minus the equivalent maturity Contant Maturity Treasury yield. All values are reported in bp. Panel C shows means (medians) for issuing firm characteristics. We allow one observation per firm, per bond type, per year. *EBITDA/AT* are the earnings before interest, taxes, and depreciation divided by total assets, $Q$ is (total assets − book equity + market equity) divided by total assets, *Leverage is* total assets − book equity divided by total assets − book equity + market equity *Equity Volatility* is the standard deviation of daily excess stock return for the prior six months, $\sigma$ *Firm Value* is the standard deviation of firm value, *Payout Ratio* is the five-year average of dividends + stock repurchases + interest expense divided by total assets – book equity + market equity.

*Panel A. Bond Characteristics*

| | Make-Whole | Noncallable | t-Test (Rank Sum) |
|---|---|---|---|
| *Rating* | 7.1 | 8.3 | 0.0001*** |
| | (7.0) | (9.0) | (0.0001) |
| *Maturity* | 11.2 | 12.0 | 0.0450*** |
| | (10.0) | (10.0) | (0.0001) |
| *Offering Amount* | $369 | $375 | 0.6361*** |
| | ($250) | (300) | (0.0006) |
| *Total Restrictive Cov* | 2.79 | 2.93 | 0.0215*** |
| | (3) | (4) | (0.0001) |
| *CMT spread* | 113 bp | 166 bp | 0.0001*** |
| | (88 bp) | (144 bp) | (0.000) |
| *FMY spread* | 9.1 bp | 16.3 bp | 0.0002*** |
| | (2.4 bp) | (7.3 bp) | (0.0010) |
| No. of obs. | 1,503 | 1,536 | |

*Panel B. Make-Whole Premiums*

| Rating | <7 Years | 7-15 Years | >15 Years |
|---|---|---|---|
| Aa-Aaa | 11.7 (12.5) | 16.8 (15) | 15.2 (15) |
| | 70.7 (66.1) | 81.7 (68.3) | 87.9 (73.9) |
| A | 17.7 (15) | 18.8 (20) | 19.4 (20) |
| | 96.0 (87.4) | 96.1 (79.0) | 105.8 (95.0) |
| Baa | 23.3 (25) | 25.7 (25) | 24.3 (25) |
| | 156.6 (139.6) | 156.2 (132.5) | 146.2 (130.9) |
| Ba | 40.4 (50) | 42.5 (50) | 41.3 (45) |
| | 249.8 (224.9) | 255.5 (237.1) | 200.2 (189.0) |
| B | 43.8 (50) | 54.2 (50) | |
| | 369.5 (367.2) | 396.9 (360.9) | – |

**Table I. Bond Characteristics, Make-Whole Premiums, and Issuing Firm Characteristics (*Continued*)**

*Panel C. Issuing Firm Characteristics*

| | Noncallable | Make-Whole | t-Test (Rank-Sum) |
|---|---|---|---|
| *EBITDA/AT* | 11.2% | 13.1% | 0.0001*** |
| | (11.0%) | (12.3%) | (0.0001) |
| *Q* | 1.55 | 1.60 | 0.1611*** |
| | (1.27) | (1.36) | (0.0009) |
| *Leverage* | 56.2% | 48.8% | 0.0001*** |
| | (54.3%) | (47.5%) | (0.0001) |
| *Equity Volatility* | 28.6% | 32.5% | 0.0001*** |
| | (25.9%) | (30.3%) | (0.0001) |
| *σ Firm Value* | 14.7% | 18.8% | 0.0001*** |
| | (13.5%) | (17.3%) | (0.0001) |
| *Payout Ratio* | 5.4% | 5.3% | 0.4654 |
| | (5.5%) | (5.4%) | (0.5308) |
| No. of obs. | 960 | 1,035 | |

*** Significant at the 0.01 level.

also calculate the spread between yield-to-maturity at issue and Bloomberg's Fair Market Yield for bonds of the same rating, maturity period, and industry group (industrial, finance, utility). Again, we use linear interpolation when there is not an exact maturity match. For the full sample of noncallable bonds, the mean (median) value is 9.1 bp (2.4 bp). For the make-whole call bonds, mean (median) FMY spreads are 16.3 bp (7.3 bp). Without additional controls, the incremental yield attributable to the make-whole call provisions in our sample appears to be approximately 5-7 bp.[11]

In Table I, Panel B, we report summary statistics for make-whole premium with bonds sorted by Moody's bond rating and maturity. To provide a comparison, we also report summary statistics on CMT spread. For the entire sample of make-whole call bonds, the mean and median make-whole premiums are approximately the same at 25 bp, with the vast majority falling between 0 bp and 50 bp. A total of nine observations have make-whole premiums greater than 50 bp with the maximum value being 75 bp. As is evident from the table, make-whole premiums increase as bond rating worsens and the bond's credit spread widens. This illustrates that make-whole premiums are not chosen randomly. Instead, make-whole premiums are usually set at 15% of the prevailing credit spread when the bond is issued (Powers and Sarkar, 2006), ensuring that the make-whole call provisions are well out-of-the-money when the bond is issued. Most of the deviation away from this thumb-rule is attributable to rounding the make-whole premium to commonly used values (e.g., 5 bp, 10 bp, 12.5 bp, 15 bp) and to capping the make-whole premium at 50 bp for noninvestment-grade debt. Regression analysis (not reported in tables) confirms that

[11]These *FMY Spreads* are similar to those reported in Mann and Powers (2003a, 2003b). However, significant caution should be exercised when evaluating these *FMY Spreads*. When calculating fair market yields, Bloomberg does not distinguish between noncallable and make-whole call bonds. This would not be an issue were it not for the fact that a majority of our noncallable bonds are in the first half of our sample period while a majority of our make-whole call bonds are in the second half of our sample period. In calculating *FMY Spread*, therefore, we tend to compare noncallable bonds to a benchmark where noncallable bonds are overrepresented and make-whole call bonds to a benchmark where make-whole call bonds are overrepresented. This is particularly true for the longer maturities where only the most recently issued bonds comprise the benchmark. Thus, the 5-7 bp incremental yield is likely to be an underestimate.

this thumb-rule for setting make-whole premiums is followed by the firms represented in our sample.

In Table I, Panel C, we present means and medians for characteristics of the firms responsible for issuing the bonds described in Panel A. In calculating these statistics, we allow only one observation per firm, per bond type in each issue year. To limit extreme outliers, all characteristics are winsorized at the first and 99th percentiles for the entire Compustat universe. Despite the weaker ratings of their bonds, make-whole call issuers are more profitable, have slightly greater perceived growth opportunities (as measured by a proxy for Tobin's *Q*), and have lower market leverage than noncallable bond issuers. The volatility of firm value, however, is greater for make-whole call issuers. Finally, payout ratio is approximately the same for both types of issuers.

# III. Model Output

## A. Calibration of Risk-Free Rate and Firm Value

To use the model, we require parameter values for the risk-free interest rate process ($r$, $\kappa_r$, $r^*$, $\sigma_r$), the asset value process ($\alpha$, $\sigma_V$, $\rho$), the capital structure of the firm ($F$, $T$, $m$, $c$), and our three financial frictions ($\lambda, z\,(V, r, T-t), \Phi, \theta$). To estimate the parameters of the risk-free process, we solve for the short-term rate $r$, the mean-reversion rate $\kappa_r$, the long-term rate $r^*$, and the volatility $\sigma_r$ that minimize the sum of squared deviations among the 1-, 2-, 3-, 5-, 10-, 20- and 30-year yields implied by the closed-form solution of the CIR model and the real-world CMT yields for the same maturities. This is done independently for each bond origination date.

For the payout ratio $\alpha$, we calculate the five-year average of (*dividends* + *stock repurchases* + *interest expense*)/(*assets* − *book value of equity* + *market value of equity*). For asset value volatility $\sigma$, we first calculate the volatility of daily equity returns $\sigma_e$ for the period beginning 360 days prior to the issuance date and ending 20 days prior to the issuance date. Asset value volatility $\sigma$ and equity volatility $\sigma$ are linked via the following relationship:

$$\sigma_e = \sigma_V \frac{V_t \partial E_t}{E_t \partial V_t}, \tag{15}$$

where $E_t$ is the market value of equity at time $t$. Following Eom, Helwege, and Huang (2004), we use the Merton (1974) model to approximate $dE_t/dV_t$ such that:

$$\sigma_e = \sigma_V * N(d_1(F, x, t))\frac{V_t}{E_t}, \tag{16}$$

where

$$d_1(F, x, t) = \frac{\ln\left(\frac{V_t}{F*P(r,t,x)}\right) + \left(-\alpha + \frac{\sigma^2}{2}\right)(x-t)}{\sigma\sqrt{x-t}}, \tag{17}$$

and $P(r,t,x)$ is the price at time $t$ of a risk-free zero-coupon bond paying \$1 at time $x$. For $\rho$, the correlation between asset value and the risk-free rate of interest, we calculate the correlation between monthly changes in asset value (*total assets* − *book value of equity* + *market value of equity*) and monthly changes in the one-year risk-free rate over the 60 months prior to the issue date of the bond.

For the capital structure of the firm, the balloon payment of the debt $F$ is set equal to the market leverage ratio of the firm: (*total assets − book value of equity*)/(*total assets − book value of equity + market value of equity*). Debt maturity $T$ is taken directly from the bond that we are analyzing—we are implicitly assuming that the bond's maturity is representative of all debt in the firm's capital structure.[12] Following Eom, Helwege, and Huang (2004), we make the proportional default cost $DC = 0.51$. If the bond has a make-whole call provision, then $m$ is set equal to the make-whole premium given in the bond's indenture.[13]

## B. Calibration of Imperfections

For the exogenous Poisson process requiring early bond retirement, we use the tender offer data of Mann and Powers (2007) to establish both the spread above the risk-free rate at which tender offers occur $z(V, r, T-t)$ as well as the intensity of the Poisson process $\lambda$. For spread $z(V, r, T-t)$, we utilize the subsample of tender offers where at least 75% of bonds were tendered. Prices of these tender offers are converted into spreads above the risk-free rate. Then, since observed tender spreads are positively correlated with credit spreads of the tendered bonds, we estimate a 10th percentile quantile regression of

$$\textit{tender spread} = a_1 * (\textit{creditspread}) + a_2 * (\textit{creditspread})^2 + \varepsilon. \tag{18}$$

A quantile regression is chosen rather than a more standard OLS estimation to ensure that we are not underestimating the value to the firm of having a make-whole call provision in these situations.[14] The results of a 10th percentile quantile regression thus correspond to a worst-case scenario where a high tender offer price (low spread above Treasuries) is necessary. The squared term is included since tender spreads level off once credit spreads are sufficiently high.

Estimated coefficients from this quantile regression are $a_1 = 0.2019$ and $a_2 = -0.000197$. These coefficient estimates are then incorporated into our model. For example, if the credit spread in our model at the time of the exogenous event is 100 bp, then we assume that the tender offer occurs at a spread of $0.2019 * 100\text{ bp} - 0.000197 * (100\text{ bp})^2 = 18.22\text{ bp}$.[15] Thus, any make-whole call provision with a premium of 18.22 bp or more will reduce the ex post wealth transferred to bondholders.

The tender offer data of Mann and Powers (2007) as well as FISD-identified transactions where the name of the purchaser includes the word "tender" are used to estimate the frequency of

[12] A potential concern is our assumption that maturity of the individual bond is representative of all of the firm's debt. We calculate a weighted-average maturity of the firm's debt listed in the FISD, excluding bank debt and commercial paper, and subtract it from the maturity of the individual bond. The differential is approximately the same for both types of bonds and has a mean (median) value of 1.17 (0.0) years. Therefore, while our simplifying assumption will induce noise in our estimates, it should not induce bias.

[13] When incorporating the make-whole call provision, there are several issues that are worth mentioning. Make-whole call provisions have a call price floor set at par value. Since in practice, coupon rates are set such that bonds initially sell at par or at a slight discount, the call price floor at par is equivalent to a call price floor set at initial bond value. If we take the coupon rate as given, our numerically solved bond values can be at a premium or discount to par. For discount bonds, setting the call price floor at par artificially decreases the states in which the call provisions are in-the-money because the floor at par binds too frequently. Conversely, for premium bonds, setting the call price floor at par artificially increases the states in which the call provisions are in-the-money because the floor at par does not bind frequently enough. To resolve this issue and to reduce bias, we use a numerical iteration procedure that finds the coupon rate $c$ for each bond such that the initial bond value is at par, that is, equal to its face value, and the call price floor is set at par value, $F$.

[14] Rather than minimizing the sum of squared residuals (as is done in OLS), a $q$th quantile regression minimizes $\Sigma_i |r_i| h_i$, where $r_i$ is the residual value for observation $i$, and $h_i = 2q$ if $r_i > 0$ or $2(1-q)$ otherwise.

[15] We also impose a cap such that, above a credit spread of 500 bp, the tender spread holds steady at 50 bp.

early redemptions. Our analysis indicates that early redemptions occur in approximately 1.6% of bond-years.[16] Hence, the annual arrival rate for the Poisson process $\lambda$ is set at 1.6%.[17]

We assume that all investors are forced to pay capital gains taxes $\tau$ of 20% on the difference between retirement price and par value whenever a call or tender offer occurs. We also assume that if the firm retires its debt, bondholders must pay transaction costs of \$1.35 per \$1,000.[18]

### C. Model-Generated Spreads

For each bond in the sample, we parameterize the stochastic processes to match observed characteristics of the risk-free yield curve, firm, and bond at the origination date. Table II displays the predicted credit spread relative to the model-generated risk-free yield for both the frictionless model and the extended model incorporating the three financial frictions. For noncallable bonds, the mean (median) predicted frictionless spread is 180 (77) bp while the with-frictions spread is 171 (72) bp.[19] For make-whole call bonds, the predicted spread includes the incremental yield associated with that bond's make-whole call provision. The predicted spread for the make-whole call bonds is significantly higher in both the frictionless environment—221 (123) bp—and in the environment with frictions—211 (117) bp.[20]

To estimate the incremental cost of the option to call, we subtract the predicted spread for an equivalent noncallable bond from the predicted spread for the make-whole call bond. In the frictionless environment, the mean (median) model-generated incremental yield for our sample is 2.08 (1.42) bp with a maximum of 19.5 bp and a minimum of 0 bp. In a frictionless capital market, the comparatively small incremental yield reflects three facts. First, in this perfect capital market setting, financial flexibility is irrelevant and the incremental yield simply reflects the pure economic value of the call.

[16]Between January 1, 1995 and December 31, 2004 (the end of our FISD transactions data) we identify 427 tender offers for noncallable and make-whole call bonds that pass the screening criteria used to initially identify our sample. These observations represent 77.3% of the known tender offers used to estimate the quantile regression. Thus, we suspect that the FISD-identified tender offers understates the true number by approximately 29.5% (1/0.773 − 1) and that the true number should be approximately 553 tender offers. In addition, we identify 68 actual calls of bonds with make-whole call provisions, making the total number of early redemptions 621 within a sample of 39,269 bond-years. Thus, we estimate that early redemptions occur in approximately 1.6% of bond-years.

[17]This arrival rate is likely to be an upper bound. For about 20% of the tender offers analyzed by Mann and Powers (2006), the tendering firm states that the reason for the tender is to refund debt or to reduce debt and interest expense. Presumably, the tendering firm only executes these tenders if the tender price is suitably low. As with the tender spread, however, we prefer to overstate the arrival rate in order to ensure that the model does not underestimate the value of a make-whole call provision.

[18]Schultz (2001) presents evidence that round-trip transactions costs for investment grade bonds average \$2.70 per \$1,000 of par value. Since early redemption proceeds are likely to be directly deposited into a bank account, we assume that investors experience one-way rather than round-trip redemption costs.

[19]On their own, capital gains taxes $\varphi$ and refunding transactions costs $\Theta$ have no impact on credit spreads for noncallable bonds since the bonds are, by definition, never retired early and the costs are never imposed. Adding the exogenous process that requires early bond retirement, however, actually leads to a reduction in noncallable bond yields since the resulting tender offer generates a windfall for investors. Capital gains taxes and refunding transactions costs slightly reduce the magnitude of this windfall.

[20]Since a make-whole call is effectively a credit spread option, correct valuation of the call option requires a model that does a reasonable job of predicting noncallable credit spreads. Our noncallable credit spreads are approximately correct on average—the average prediction error (model-generated credit spread minus observed credit spread) for the noncallable bonds is 67 bp while the median prediction error is −18 bp. As with most structural models, there is significant cross-sectional variation in accuracy. However, the variation is of similar magnitude to results presented in prior research where the predictive ability of various structural models are tested (see, e.g., Jones, Mason, and Rosenfeld, 1984; Wei and Guo, 1997; Lyden and Saranti, 2000; Eom, Helwege, and Huang, 2004). Details are available from the authors by request.

## Table II. Model-Generated Credit Spreads

*Credit Spread* is the model-generated yield-to-maturity minus the model-generated risk-free rate. For make-whole call bonds, *Predicted Spread* incorporates the inclusion of a make-whole call provision. *Incremental Yield* is reported only for make-whole call bonds and is the *Predicted Spread* for the make-whole call bond minus the *Predicted Spread* for a noncallable bond that, with the exception of the make-whole call provision, is equivalent in all respects. Results are reported both for a frictionless modeling environment and for an environment where the three financial frictions are incorporated into the model.

| | Noncallable | Make-Whole |
|---|---|---|
| Frictionless environment | | |
| *Credit Spread* | 180 bp | 221 bp |
| | (77 bp) | (123 bp) |
| *Incremental Yield* | | 2.08 bp |
| | | (1.42 bp) |
| Environment with frictions | | |
| *Credit Spread* | 171 bp | 211 bp |
| | (72 bp) | (117 bp) |
| *Incremental Yield* | | 5.40 bp |
| | | (3.72 bp) |

Second, the firm will call only when it can profitably refund the existing bond with an equivalent bond at lower cost. For this to occur, the credit spread on new borrowing for the firm must be less than the make-whole premium and the market price of the bond must be above par (or else the call price floor at par is binding). As previously noted, however, make-whole premiums are typically set at 15% of the when-issued credit spread. Given the small magnitude of make-whole premiums, the likelihood of a profitable refunding opportunity is small.[21] Finally, the small incremental yield reflects that the pay-off to the firm from exercising the call is capped.[22]

In the environment with frictions, the mean (median) model-generated incremental yield is 5.40 (3.72) bp. While incremental yields increase when frictions are included in the model, model-generated incremental yields are still relatively low since exogenous events requiring early retirement are relatively infrequent in practice and the alternative method for retiring debt early, that is, a tender offer, is often not particularly expensive for the firm relative to the calculated make-whole call price.

[21] While potential in-the-money calls will account for some of the real-world incremental yield, our model likely overstates the effect. This is because our model is strictly a default model where credit spreads can potentially narrow to zero. In practice, however, corporate credit spreads incorporate other factors such as liquidity risk, systematic risk, and differential state taxation relative to Treasury securities (Elton et al., 2001). Thus, while credit spreads in our model can theoretically be close to zero, these additional factors lend an almost permanent additional component to observed credit spreads. This ensures that even the most creditworthy corporate bonds have yields that are significantly greater than risk-free yields. Since 1954, for example, the credit spread for seasoned Aaa corporate bonds has averaged 75 bp. In this environment, the typical make-whole call provision with a make-whole premium of 25 bp will have minimal economic value since it is almost always out-of-the-money.

[22] If the make-whole premium is 25 bp, the maximum that the firm can save when refunding is 25 bp of yield. For a bond with five years until maturity and a 7% coupon, discounting at 6.75% rather than at 7% implies a savings of $10.46 per $1,000 of face value.

# IV. Empirical Analysis of Incremental Yields

To measure actual incremental yields, we estimate OLS regressions using two distinctly different measures of spreads. The first equation is

$$CMT\ Spread = a_0 + a_1 * MW + a_2' * Default\ Proxies + a_3' * (Controls) + \varepsilon. \quad (19)$$

Here, *CMT Spread* is the spread in bp over the same maturity Treasury, *MW* is a dichotomous variable that is coded 1 for make-whole call bonds, *Default Proxies* are the model inputs that describe default-related characteristics of the issuing firm and the bond, and *Controls* are a variety of factors that previous research has shown are related to credit spreads but which are not incorporated in our structural model. The second equation that we estimate is

$$FMY\ Spread = a_0 + a_1 * MW + a_2' * (Controls) + \varepsilon. \quad (20)$$

Since *FMY Spread* is the difference of bond yield and Bloomberg's Fair Market Yield for bonds of the same rating, maturity, and major industry, there is no need to include *Default Proxies*. In both regressions, we winsorize the dependent variable at the first and 99th percentiles to limit the affect of outlying observations.

An advantage of Equation (19) is that we can observe the explanatory power of the basic inputs in our model. In addition, we do not have to rely on the "black box" of the ratings agencies. Conversely, an advantage of Equation (20) is that we make direct use of ratings agency expertise—default risk is removed from consideration by subtracting Bloomberg's Fair Market Yield when forming the dependent variable. Moreover, Bloomberg's Fair Market Yield implicitly incorporates nondefault characteristics of corporate bonds that affect credit spreads, such as low liquidity relative to Treasuries. A disadvantage of Equation (20), however, is that, as noted in Section II, Bloomberg's Fair Market Yield might be a biased measure of the default component of credit spreads.

## A. Control Variables

Before reporting the regression results, we describe the control variables. Throughout the sample period, there is wide variation in credit spreads.[23] When estimating Equation (19), we account for exogenous variation in the credit environment across time periods by calculating the issuance day spread between the same rating, same-maturity Bloomberg Fair Market Yield and same-maturity Treasury yield. We then calculate the average daily value for this spread over the period of our sample. Subtracting the average spread from the issuance day spread gives us a variable *Spread Deviation* that controls for exogenous credit spread fluctuations. In Equation (20), *Spread Deviation* is not necessary since variation in credit spreads is already incorporated in the dependent variable *FMY Spread*.

To control for bond liquidity, we include *Log Offering Amount* and *Log Maturity* (Chen, Lesmond, and Wei, 2007). Including the log of maturity is also motivated by the fact that maturity is an input in our model and research indicates that the observed term structure of credit spreads widens with maturity (Litterman and Iben, 1991; Fons, 1994; Duffee, 1998). Since utilities and financial service firms are often regarded as distinctly different credit risks (Elton, Gruber,

[23]During the sample period, there was a "flight to quality" during the Asian debt crisis of 1997 and the long-term capital management (LTCM) and Russian debt crises of 1998.

Agrawal, and Mann, 2001; Campbell and Taksler, 2003), we include *Finance* and *Utility* that are coded 1 whenever the issuer is a financial services firm or a regulated utility.

Since credit spreads for bonds issued privately under Rule 144a are significantly greater than for comparable publicly issued bonds (Fenn, 2000; Livingston and Zhou, 2002; Chaplinsky and Ramchand, 2004), *Rule 144a* is coded 1 whenever the bond is privately issued. To control for higher spreads for first-time bond issuers (Datta, Iskandar-Datta, and Patel, 1997), *First Bond* is coded 1 whenever no earlier bond issued by the same firm is recorded in the FISD. We also include a dichotomous variable *Senior* that is coded 1 if the bond is not designated as subordinated.[24]

Since issues underwritten by more reputable investment banks have lower initial day returns and thus less underpricing (Datta, Iskandar-Datta, and Patel, 1997), and since more reputable underwriters are also associated with tighter at-issue credit spreads (Livingston and Miller, 2000), we measure and control for underwriter prestige by calculating the percentage of total corporate bond offerings lead by that particular investment bank over the prior five years.[25] Finally, Bradley and Roberts (2004) demonstrate that, after controlling for endogeneity, bonds with restrictive covenants are issued at lower credit spreads than bonds without restrictive covenants. To control for possible differences in covenants we include the number of restrictive covenants present in each bond.

### B. Regression Results

Results from OLS estimation of Equations (19) and (20) are displayed in Table III. All $t$-statistics are calculated using robust standard errors with clustering by issuer. In each specification, the signs of estimated coefficients for the exogenous control variables are generally as expected and, in most cases, values are statistically significant. At-issue credit spreads increase with *Spread Deviation*. Credit spreads are greater for privately issued bonds, longer maturity bonds, and for bonds that represent the first debt issue of the firm.

Credit spreads are lower when the offering amount is large. Surprisingly, bonds with a greater number of restrictive covenants have higher rather than lower spreads.[26] Bonds issued by firms in the financial industry have lower credit spreads. Finally, estimated coefficients for underwriter prestige and for whether the bond was issued by a utility or was designated senior are not statistically significant.

Estimated coefficients for the default proxies used as model inputs are generally as expected in Specification 1. Credit spreads increase with leverage, the volatility of firm value, and with payout ratio. The estimated coefficient for the correlation between firm value and interest rates is not significantly different from zero while the coefficient for the risk-free rate is negative.

Of greatest importance are the results for the dichotomous *MW* indicator. In specification 1 where the dependent variable is *CMT Spread,* the estimated coefficient for *MW* is 16.5 bp with a $t$-statistic of 4.58. In Specification 2 where the dependent variable is *FMY Spread,* the estimated coefficient for *MW* is 13.6 bp with a $t$-statistic of 4.66. Unreported $t$-tests show that both values are statistically significantly greater than the average of 2.1 bp predicted by our frictionless

[24] Financial service firms and regulated utilities comprise 28.1% and 8.3% of the sample. Privately issued bonds are 15.4% of our sample while first bonds are 15.9% of our sample. Finally, 93.6% of the bonds are not subordinated. Make-whole call bonds are significantly more likely than noncallable bonds to be issued under Rule 144a (20.9% vs. 9.6%) and to be the first bond issued by the firm (20.6% vs. 11%).

[25] For our noncallable sample, *Underwriter Prestige* averages 6%. For our make-whole call sample, the average is 6.8%.

[26] In all likelihood, this reflects endogeneity in the decision to incorporate these features (Bradley and Roberts, 2004).

## Table III. OLS Regression of Credit Spreads

In Specification 1, the dependent variable is the bond's credit spread at issue. In Specification 2, the dependent variable is yield at issue minus Bloomberg's Fair Market Yield. Independent variables are as follows. *MW* is coded 1 for make-whole call bonds, 0 otherwise. *Leverage* is the total assets – book equity divided by total assets – book equity + market equity. *Volatility* is the standard deviation of firm value. *Payout Ratio* is 5-year average of dividends + stock repurchases + interest expense divided by total assets – book equity + market equity. $Correlation_{V-Rf}$ is the correlation between firm value and 10-year Constant Maturity Treasury yield calculated for the industry of the issuer. *Risk-Free Rate* is the 10-year Constant Maturity Treasury yield. *Spread Deviation* is the deviation of Bloomberg's Fair Market Yield minus the Constant Maturity Treasury rate from its long-term average. *Rule 144a* is coded 1 for privately issued bonds, 0 otherwise. *Finance* and *Utility* are coded 1 for financial services and utility firms, respectively, 0 otherwise. *Log Maturity* and *Log Offering Amount* are the logs of time-to-maturity in years and nominal face value, respectively. *First Bond* is coded 1 when the observation represents the first bond issue for the firm, 0 otherwise. *Senior* is coded 1 if the bond is denoted senior or senior-secured, 0 otherwise. *Restrictive Covenants* is the number of restrictive covenants incorporated in the bond indenture. *Underwriter Prestige* is the percentage of total corporate bond offerings in the past five years underwritten by the lead underwriter. Below each coefficient estimate is the robust *t*-statistic adjusted for clustering of observations by issuer.

| | Specification 1 | Specification 2 |
|---|---|---|
| Constant | 74.784** | 26.568 |
| | (2.05) | (1.19) |
| *Make-Whole* | 16.513*** | 13.631*** |
| | (4.58) | (4.66) |
| *Leverage* | 170.76*** | |
| | (11.58) | |
| *Volatility* | 414.19*** | |
| | (10.73) | |
| *Payout Ratio* | 1,292.1*** | |
| | (9.09) | |
| $Correlation_{V-Rf}$ | 33.029** | |
| | (2.37) | |
| *Risk-Free Rate* | −9.219*** | |
| | (4.09) | |
| *Spread Deviation* | 0.603*** | |
| | (12.46) | |
| *Rule 144A* | 67.433*** | 23.111*** |
| | (9.55) | (3.73) |
| *Finance* | −10.895* | 15.253*** |
| | (1.79) | (4.02) |
| *Utility* | −5.429 | −3.450 |
| | (0.66) | (0.65) |
| *Log Maturity* | 8.801*** | −4.159*** |
| | (4.69) | (2.67) |
| *Log Offering Amount* | −12.888*** | −2.120*** |
| | (5.67) | (1.29) |
| *First Bond* | 15.036*** | 7.438** |
| | (3.74) | (2.05) |
| *Senior* | −6.872*** | 14.084*** |
| | (0.95) | (3.33) |

**Table III. OLS Regression of Credit Spreads (*Continued*)**

| | Specification 1 | Specification 2 |
|---|---|---|
| *Restrictive Covenants* | 9.861** | 1.522 |
| | (6.70) | (1.24) |
| *Underwriter Prestige* | −7.303 | 3.250 |
| | (1.13) | (0.67) |
| No. of obs. | 3,029 | 2,939 |
| Adj. $R^2$ | 0.5540 | 0.0676 |

*** Significant at the 0.01 level.
** Significant at the 0.05 level.
* Significant at the 0.10 level.

model and 5.4 bp predicted by our model with financial frictions. Thus, two distinctly different estimation methods both suggest that investors demand almost three times the compensation that our model indicates should be offered for the additional risk associated with a make-whole call provision. Given that our model with frictions is deliberately calibrated to provide an upper bound on theoretical incremental yields, these results are surprising and leave us puzzled.

What explains the fact that observed incremental yields seem so large compared to the predictions of our model? One possibility is that our structural model might be misspecified. Our analysis, however, is based on comparing bonds with make-whole call provisions to their non-callable equivalents. Any deficiencies in the model vis-à-vis the default component of credit spreads, therefore, are likely to be neutralized via this "relative approach," particularly since the distribution of model inputs is approximately the same for both types of bonds. Alternatively, vital inputs may be missing from our model. In our empirical analysis, however, we control for a wide variety of additional factors that have been shown to affect credit spreads but that are not typically incorporated in structural models. Another possible explanation for the magnitude of our results is that the decision to incorporate a make-whole call provision may be endogenous. In the following section we analyze possible endogeneity in detail.

## V. Robustness Issues

### A. Potential Endogeneity and Adverse Selection

To control for endogeneity, we replicate the two specifications presented in Table III using a treatment effects regression as described by Maddala (1983). The first stage is a probit regression where *MW* is the dependent variable. The second stage is OLS augmented by a hazard ratio from the first stage as a control for potential endogeneity bias. The actual estimation is straightforward, provided that adequate exogenous instruments are available for the first-stage probit regression.

Several of our instruments reflect the increase in popularity of make-whole call provisions over time as displayed in Figure 1. First, underwriters gradually became familiar with make-whole call provisions with some underwriters gaining familiarity earlier than others. Since issuers are likely to be influenced by advice from their underwriter, we calculate and utilize as an instrument the percentage of bonds the lead underwriter underwrote in the previous calendar quarter that contained make-whole call provisions. Second, once familiarized with make-whole call provisions, firms are more likely to include them in subsequent offerings regardless of the recommendations of the underwriter. Thus, we include a dichotomous variable that is coded 1 if any of the bonds offered by the firm in its most recent previous bond offering had make-whole call provisions. Third, since younger firms are likely to be more innovative in structuring securities, we include a dichotomous variable that is coded 1 if the bond we are analyzing is the first bond issued by the issuing firm. Finally, to control for the general increase in popularity of make-whole call provisions over time, we include a time variable equal to year of issue minus 1995.

Since the financial flexibility provided by a make-whole call provision is more valuable the longer the maturity of the bond, we include log of bond maturity as a control in the first-stage regression. As our last instrumental variable, we include a dichotomous variable that is coded 1 for issuers in the financial services industry.

Results of the first-stage probit regressions are presented in Table IV, Panel A. All of the estimated coefficients for our instruments are statistically significant and, with the exception of the financial services variable, have positive signs. Thus, a firm is more likely to incorporate a make-whole call provision if the underwriter has significant experience underwriting make-whole callable bonds, if the firm recently issued make-whole callable bonds, and so on.

Results for the second-stage OLS regression are presented in Table IV, Panel B. With the exception of *MW*, the treatment effect has minimal impact on all coefficient estimates. For the *MW* indicator, however, we see an increase in coefficient estimates once endogeneity is accounted for. In the first specification, for example, the coefficient estimate increases from 16.5 bp to 24.6 bp ($t$-statistic = 3.05). In the second specification, the coefficient estimate increases from 13.6 bp to 18.4 bp ($t$-statistic = 3.35).[27]

The increase in magnitude of the estimated coefficients after accounting for endogeneity is not surprising. Consider two firms, a risky firm that must offer a high yield on new bonds and a less risky firm that can issue bonds with lower yields. Our belief is that the high-risk/high-yield firm would be less likely to incorporate a make-whole call provision, all else equal, since this would force the firm to offer an even higher yield on its bonds. Thus, risk-related endogeneity, if present, is likely to generate a downward bias in our estimated coefficients. Alternatively, assume the typical firm issuing a make-whole call bond has private information about its default risk. Given the structure of a make-whole call provision, it is firms having private information that their default risk is lower than the default risk predicted by the market that would benefit most from incorporating a make-whole call provision. Thus, if adverse selection is a concern, it should again generate a downward bias in estimated coefficients for the *MW* variable if controls for endogeneity are lacking.

There could also be unobserved differences in the need for the financial flexibility offered by a make-whole call provision. Firms with the greatest need for financial flexibility should be those that are likely to experience events requiring early bond retirement. Since early retirement necessitates paying a premium to bondholders (whether retirement is via a tender offer or make-whole call provision), credit spreads for firms requiring flexibility should be lower. Thus, if make-whole call provisions are predominantly utilized by firms with an unobserved need for financial flexibility, we should observe lower credit spreads and again experience a downward bias in our estimated coefficients for the *MW* variable if endogeneity is not taken into account.

### B. The Time Pattern of Incremental Yields

To examine whether there is any change in the valuation of make-whole call provisions across time, we split our sample into three time periods: early (1995-1998), mid (1999-2001), and

[27] A Wald test for whether the coefficient estimates are significantly different after accounting for endogeneity is marginally significant for specification one ($p$-value = 0.069) but is insignificant for specification two ($p$-value = 0.279).

## Table IV. Treatment Effects Regression of Credit Spreads: First Stage and Second Stage

In Panel A, the first stage in the treatment effects regression is a probit regression where *MW* (coded 1 for make-whole call bonds, 0 otherwise) is the dependent variable. In Specification 1, the dependent variable is the bond's credit spread at issue. In Specification 2, the dependent variable is yield at issue minus Bloomberg's Fair Market Yield. Independent variables are as follows. *Prior MW* and *Prior Fixed-Price* are coded 1 if any of the bonds in the most recent previous bond offering by the firm had a make-whole call provision or had a fixed-price call provision, 0 otherwise. *First Bond* is coded 1 when the observation represents the first bond issue for the firm, 0 otherwise. *Underwriter Frequency* is the percentage of bond issues in the previous calendar quarter that were underwritten by the lead underwriter and that had a make-whole call provision. *Time* is year of offering date minus 1995. *Log Maturity* is the log of time-to-maturity in years. *Finance* is coded 1 for financial services firms, 0 otherwise. Below each coefficient estimate is the robust $z$-statistic adjusted for clustering of observations by issuer. In Panel B, the second stage in the treatment effects regression is OLS where a hazard function from the first stage is included to control for potential endogeneity of *MW*. In Specification 1, the dependent variable is the bond's credit spread at issue. In Specification 2, the dependent variable is yield at issue minus Bloomberg's Fair Market Yield. Independent variables are as follows. *MW* is coded 1 for make-whole call bonds, 0 otherwise. *Leverage* is total assets − book equity divided by total assets − book equity + market equity. *Volatility* is the standard deviation of firm value. *Payout Ratio* is the five-year average of dividends + stock repurchases + interest expense divided by total assets − book equity + market equity. $Correlation_{V-Rf}$ is the correlation between firm value and 10-year Constant Maturity Treasury yield calculated for the industry of the issuer. *Risk-Free Rate* is the 10-year Constant Maturity Treasury yield. *Spread Deviation* is the deviation of Bloomberg's Fair Market Yield minus the Constant Maturity Treasury rate from its long-term average. *Rule 144a* is coded 1 for privately issued bonds, 0 otherwise. *Finance* and *Utility* are coded 1 for financial services and utility firms, respectively, 0 otherwise. *Log Maturity* and *Log Offering Amount* are the logs of time-to-maturity in years and nominal face value respectively. *First Bond* is coded 1 when the observation represents the first bond issue for the firm, 0 otherwise. *Senior* is coded 1 if the bond is denoted senior or senior-secured, 0 otherwise. *Restrictive Covenants* is the number of restrictive covenants incorporated in the bond indenture. *Underwriter Prestige* is the percentage of total corporate bond offerings in the past five years underwritten by the lead underwriter. Below each coefficient estimate is the robust $t$-statistic adjusted for clustering of observations by issuer.

*Panel A. First Stage*

| | Specification 1 | Specification 2 |
|---|---|---|
| Constant | −2.308*** | −2.417*** |
| | (14.17) | (15.54) |
| *Prior MW* | 1.387*** | 1.396*** |
| | (13.15) | (13.14) |
| *Prior Fixed-Price* | 0.340*** | 0.329*** |
| | (2.84) | (2.76) |
| *First Bond* | 1.145*** | 1.159*** |
| | (11.47) | (11.48) |
| *Underwriter Frequency* | 0.778*** | 0.715*** |
| | (4.51) | (4.18) |
| *Time* | 0.143*** | 0.156*** |
| | (6.59) | (7.29) |
| *Log Maturity* | 0.430*** | 0.459*** |
| | (7.99) | (8.97) |
| *Finance* | −0.680 | −0.714*** |
| | (6.46) | (6.66) |

**Table IV. Treatment Effects Regression of Credit Spreads: First Stage and Second Stage (*Continued*)**

*Panel B. Second Stage*

| | Specification 1 | Specification 2 |
|---|---|---|
| Constant | 72.089* | 29.686 |
| | (1.94) | (1.31) |
| *MW* | 24.559*** | 18.355*** |
| | (3.05) | (3.35) |
| *Leverage* | 169.35*** | |
| | (11.58) | |
| *Volatility* | 411.09*** | |
| | (10.59) | |
| *Payout Ratio* | 1,275.6*** | |
| | (8.81) | |
| *Correlation*$_{V-Rf}$ | 30.144** | |
| | (2.16) | |
| *Risk-Free Rate* | −8.223*** | |
| | (3.41) | |
| *Spread Deviation* | 0.606*** | |
| | (11.91) | |
| *Rule 144A* | 66.263*** | 21.306*** |
| | (9.18) | (3.40) |
| *Finance* | −7.577 | 15.945*** |
| | (1.18) | (3.93) |
| *Utility* | −4.381 | −4.571 |
| | (0.53) | (0.84) |
| *Log Maturity* | 8.292*** | −4.787*** |
| | (4.23) | (2.96) |
| *Log Offering Amount* | −13.157*** | −2.254 |
| | (5.72) | (1.33) |
| *First Bond* | 13.373*** | 6.813* |
| | (2.99) | (1.78) |
| *Senior* | −6.385 | 14.102*** |
| | (0.85) | (3.18) |
| *Restrictive Covenants* | 9.428*** | 1.316 |
| | (6.17) | (1.07) |
| *Underwriter Prestige* | −5.832 | 1.658 |
| | (0.88) | (0.32) |
| No. of obs. | 2,910 | 2,827 |

*** Significant at the 0.01 level.
** Significant at the 0.05 level.
* Significant at the 0.10 level.

late (2002-2004). This split allocates approximately the same number of make-whole call bonds to each period. In Table V, we present OLS estimation results incorporating five dichotomous variables: *Mid*, *Late*, *MW_Early*, *MW_Mid*, and *MW_Late*. The last three variables denote make-whole call bonds issued during each time period. All of the additional control variables used in Tables III and IV are included in the regression, but their estimated coefficients are not displayed

## Table V. Time Pattern of Observed Credit Spreads

In Specification 1, the dependent variable is the bond's credit spread at issue. In Specification 2, the dependent variable is yield at issue minus Bloomberg's Fair Market Yield. Independent variables are as follows. *Early* and *Mid* are dichotomous variables corresponding to the subperiods 1993-1998 and 1999-2001. Similarly, *MW_Early, MW_Mid,* and *MW_Late* are the make-whole call dichotomous variables for the three respective subperiods. All of the additional control variables utilized in Tables III and IV are also included here; however, results are not displayed for these variables in order to conserve space. The additional control variables are *Spread_Deviation, Rule 144a, Finance, Utility, Log Maturity, Log Offering Amount, First Bond, Senior, Restrictive Covenants,* and *Underwriter Prestige.* Each is described in the earlier tables. Below each coefficient estimate is the *t*-statistic adjusted for clustering of observations by issuer.

| | **Specification 1** | **Specification 2** |
|---|---|---|
| Constant | 88.880** | 46.971* |
| | (2.27) | (1.73) |
| *Early* | 1.074 | 3.582 |
| | (0.12) | (0.46) |
| *Mid* | 5.661 | 11.824 |
| | (0.64) | (1.47) |
| *MW_ Early* | 23.832*** | 11.470*** |
| | (5.32) | (3.46) |
| *MW_ Mid* | 16.327*** | 18.286*** |
| | (2.79) | (3.73) |
| *MW_ Late* | 4.584 | 3.543 |
| | (0.40) | (0.58) |
| No. of obs. | 3,029 | 2,939 |
| Adj. $R^2$ | 0.5541 | 0.0822 |

*** Significant at the 0.01 level.
** Significant at the 0.05 level.
* Significant at the 0.10 level.

in order to conserve space. The estimated early, mid, and late *MW* coefficients are 23.8 bp, 16.3 bp, and 4.6 bp in specification 1. In Specification 2, the estimated early, mid, and late *MW* coefficients are 11.5 bp, 18.3 bp, and 3.5 bp. Neither of the late-period *MW* coefficient estimates is statistically significant. However, all of the early- and mid-period *MW* coefficients are statistically significant. The decline in estimated coefficients, particularly between 1999 and 2001 and between 2002 and 2004 suggests that incremental costs of make-whole call provisions are gradually converging to the values predicted by our model.

Greater make-whole premiums in the early half of the sample period could potentially explain the higher *MW* coefficient estimates in the 1995-1998 and 1999-2001 time periods. In actuality, however, make-whole call premiums are slightly larger later in the sample period, whether measured unconditionally or relative to the credit spread of the bond at-issue. An alternative explanation for the higher *MW* coefficient estimates in the 1995-1998 and 1999-2001 time periods is that, in the early 1990s just prior to the introduction of publicly traded make-whole call bonds in the United States, investors became averse to call provisions in general. As noted by King and Mauer (2000), there was a significant increase in call activity for bonds with fixed-price call provisions that coincided with the sharp decline in interest rates that occurred in 1986. This increase in call activity reached a peak in the early 1990s. A majority of these calls were likely driven by firms refunding outstanding debt with new lower interest rate debt. Hence, holders of the called bonds were at a disadvantage as they were forced to reinvest call proceeds at the lower prevailing interest rates. Arguably, this negative experience reduced the attractiveness to investors of bonds with any type of call provision, even though make-whole call provisions are not subject to interest rate risk. Over time, one would expect investors to learn about the distinctive characteristics of make-whole call provisions and begin to value them appropriately. The decline, over time, in the coefficient estimates for the *MW* indicator is consistent with this story.

## VI. Conclusion

Options attached to corporate bonds, whether they are fixed-price call provisions, make-whole call provisions, put provisions, or conversion options, entail benefits and costs for both the issuing firm and bondholders. In the case of a call provision, the issuing firm gains the ability to call the bond when economic conditions change significantly and the call provision is in-the-money. In addition, the firm gains the flexibility to call the bond when firm-specific events require that the firm retire the bond early. Since bondholders are short the call option, they require compensation in the form of additional (incremental) yield. A portion of this incremental yield will be attributable to the value that bondholders expect will be expropriated from them when the firm calls the bond. The remainder should be attributable to market imperfections that impose deadweight costs on bondholders when calls occur.

When we apply our structural model to a large sample of recently issued bonds with make-whole call provisions, we find that in a frictionless market where calls only occur when the call is in-the-money, the incremental yield required by bondholders should average only about 2 bp. When imperfections, such as capital gains taxes, transactions costs, and randomly arriving exogenous events that require early retirement, are incorporated into our model, the incremental yield increases to slightly more than 5 bp.

When we decompose observed at-issue credit spreads into their default cost component and the incremental yield attributable to the make-whole call provision, we find that observed incremental yields are significantly greater than what is predicted by either our frictionless or our extended model. Regression results indicate that observed incremental yields average between 13 and 24 bp.

Survey evidence in Mann and Powers (2003b) makes it clear that corporate executives believe that make-whole call provisions offer tangible benefits to the firm in the form of increased financial flexibility. Thus, firms are willing to pay a premium in order to incorporate make-whole call provisions. In a competitive market, however, the incremental yield of a make-whole call provision should reflect the costs imposed on the marginal investor, not the benefits gained by the firm. This is particularly true if the benefits to the firm of increased financial flexibility do not translate one-for-one into costs imposed on investors. Fortunately, it appears that this equilibrium is gradually occurring. While make-whole call provisions have been significantly overpriced on average across our 10-year sample, observed incremental yields during the last three years of our sample appear to be significantly less than observed incremental yields for the first four years of our sample. If the predictions of our model are valid and competitive forces continue to act, we should continue to see lower values for make-whole call provision incremental yields. Whether this prediction holds true remains to be seen.■

## Appendix: Valuation of Equity and Debt for the Model with Imperfections

We use the following notation for the model that incorporates imperfections: $\lambda$ is the intensity of the Poisson arrival of exogenous events that induce the firm to retire debt early. Thus, $\lambda .dt$ is the probability that the firm will have to retire its bond for exogenous reasons during the period $dt$. Transaction costs and the tax rate are denoted by $\theta$ and $\phi$, respectively.

As in the frictionless economy, at the debt maturity date $T$, the value of the firm's equity is

$$E(V, r, T) = \max(V - F, 0). \tag{A1}$$

The expression in Equation (9) for the value of equity $E$ any time prior to maturity, $t < T$, is modified. In the economy with frictions, the equity value depends on the arrival of exogenous events and is given as the solution to the following PDE:

$$\begin{aligned}&\tfrac{\sigma_r^2 r}{2} E_{rr} + \tfrac{\sigma_V^2 V^2}{2} E_{VV} + \rho\sigma_V\sigma_r V\sqrt{r}E_{rV} + \kappa_r(r^* - r)E_r + (r-\alpha)E_V \\ &\quad + E_t + \alpha V - c + \lambda\{V - E - B(V, r, T-t)\} - rE = 0, \quad E \geq 0,\end{aligned} \tag{A2}$$

where $B(V, r, T-t)$ is the price that the firm pays for the bond with $T-t$ remaining until maturity in case an exogenous event arrives.

When an exogenous event arrives, the firm with a noncallable bond tenders for its bond at the risk-free rate plus a calculated tender spread $z(V, r, T-t)$. As described in Section III.B, the tender spread is a function of the spread between the bond's current yield-to-maturity and the yield-to-maturity of the equivalent risk-free bond. As described in Section III.B, the tender spread $z(V, r, T-t)$ in bp is calculated as follows:

$$z(V, r, T-t) = 0.2019 \times (\text{market spread in bp}) - 0.000197 \times (\text{market spread in bp})^2.$$

Thus, for noncallable bonds, the tender price is given by

$$B(V, r, T-t) = \int_t^T ce^{-z(V,r,T-t)(x-t)}P(r, t, x)dx + F \cdot P(r, t, T)e^{-z(V,r,T-t)(T-t)}, \tag{A3}$$

where $P(r, t, x)$ is the price of a risk-free zero-coupon bond paying \$1 at time $x$, calculated according to the CIR formula featured in Equation (4). The last two terms in Equation (A3) are the present values of the remaining coupon payments and of the balloon payment discounted at the risk-free rate plus the spread $z$.

At the arrival of an exogenous event, the firm that has a make-whole call provision retires the debt at the minimum of tender price or make-whole call price, that is, $\min[B(V, r, T-t), M(r, c, F, m, T-t)]$. Equivalently, the firm retires its debt at the maximum of tender spread or make-whole call premium, that is, $\max[z(V, r, T-t), m]$. Given the expression for the make-whole call price in Equation (3) for bonds with make-whole call provisions, the price of early retirement due to an exogenous event is

$$B(V,r,T-t) \\ = \int_t^T ce^{-\max[z(V,r,T-t),m](x-t)} P(r,t,x)dx + F \cdot P(r,t,T)e^{-\max[z(V,r,T-t),m](T-t)}. \quad \text{(A4)}$$

As in the frictionless model, there are additional free boundary conditions that characterize cases where the firm with a make-whole call bond retires its bond endogenously, and where the firm defaults. The endogenous call boundary (if applicable) should satisfy

$$E(V,r) > V - M(r,c,F,m,T-t).$$

To calculate the value of the debt $D$, we need to consider the firm's exogenous and endogenous (if applicable) calls and default. As in the case without frictions, at maturity $T$, the debt value is given by

$$\begin{cases} D(V,r,T) = F, \text{ if } V \geq F \\ D(V,r,T) = (1-DC) \cdot V, \text{ otherwise.} \end{cases}$$

Prior to maturity, the PDE in Equation (12) for the value of debt is modified and satisfies:

$$\frac{\sigma_r^2 r}{2} D_{rr} + \frac{\sigma_V^2 V^2}{2} D_{VV} + \rho\sigma_V\sigma_r V\sqrt{r} D_{rV} + \kappa_r(r^* - r)D_r + (r-\alpha)D_V + D_t \\ + \lambda\{B(V,r,T-t) - D - \theta \cdot B(V,r,T-t) - \phi \cdot (B(V,r,T-t) - F)\} - rD + c = 0, \quad \text{(A5)}$$

where the terms $\theta \cdot B(V, r, T-t)$ and $\phi \cdot (B(V, r, T-t)-F)$ are transactions costs and taxes, respectively, that debtholders pay if the firm retires the bond. Taxes are imposed on the difference between the price at which the firm retires the bond and the bond's face value.

Similar to the frictionless case, for a callable bond, the boundary condition that corresponds to the case where the firm buys the debt back at the make-whole call price or at the tender price is

$$D(V,r,t) = B(r,c,F,m,T-t) - \theta \cdot B(r,c,F,m,T-t) - \Phi \cdot (B(r,c,F,m,T-t) - F), \\ if\ E(V,r,t) = V - B(r,c,F,m,T-t),$$

where the last two terms are transaction costs and taxes incurred by the debtholders. Similarly, the boundary condition for the value of debt when equity is valueless and the firm defaults is

$$D(V,r,t) = (1-DC) \cdot V, \quad \text{if} \quad E(V,r,t) = 0.$$

## References


Acharya, V.V. and J. Carpenter, 2002, "Corporate Bond Valuation and Hedging with Stochastic Interest Rates and Endogenous Bankruptcy," *Review of Financial Studies* 15, 1355-1383.

Bradley, M. and M.R. Roberts, 2004, "The Structure and Pricing of Corporate Debt Covenants," Duke University Working Paper.

Campbell, J.Y. and G.B. Taksler, 2003, "Equity Volatility and Corporate Bond Yields," *Journal of Finance* 58, 2321-2350.

Chaplinsky, S. and L. Ramchand, 2004, "The Impact of SEC Rule 144A On Corporate Debt Issuance by International Firms," *Journal of Business* 77, 1073-1097.

Chen, L., D.A. Lesmond, and J. Wei, 2007, "Corporate Yield Spreads and Bond Liquidity," *Journal of Finance* 62, 119-149.

Cox, J., J. Ingersoll, and S. Ross, 1985, "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica* 53, 385-407.

Datta, S., M. Iskandar-Datta, and A. Patel, 1997, "The Pricing of Initial Public Offers of Corporate Straight Debt," *Journal of Finance* 52, 379-396.

Duffee, G.R., 1998, "The Relation between Treasury Yields and Corporate Bond Yield Spreads," *Journal of Finance* 53, 2225-2241.

Elton, E.J., M.J. Gruber, D. Agrawal, and C. Mann, 2001, "Explaining the Rate Spread on Corporate Bonds," *Journal of Finance* 56, 247-277.

Eom, Y.H., J. Helwege, and J.Z. Huang, 2004, "Structural Models of Corporate Bond Pricing: An Empirical Analysis," *Review of Financial Studies* 17, 499-544.

Fenn, G.W., 2000, "Speed of Issuance and the Adequacy of Disclosure in the 144a High-Yield Debt Market," *Journal of Financial Economics* 56, 383-405.

Fons, J.S., 1994, "Using Default Rates to Model the Term Structure of Credit Risk," *Financial Analysts Journal* 50, 25-32.

Graham, J.R., 2000, "How Big Are the Tax Benefits of Debt?" *Journal of Finance* 55, 1901-1941.

Graham, J.R. and C. Harvey, 2001, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60, 187-243.

Güntay, L., N. Prabhala, and H. Unal, 2004, "Callable Bonds, Interest-Rate Risk, and the Supply Side of Hedging," University of Maryland Working Paper.

Jacoby, G. and D. Stangeland, 2004, "The Make-Whole, Doomsday, and Canada Call Provisions," University of Manitoba Working Paper.

Jones, E.P., S.P. Mason, and E. Rosenfeld, 1984, "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Analysis," *Journal of Finance* 39, 611-625.

Kahan, M. and B. Tuckman, 1993, "Private vs. Public Lending: Evidence from Covenants," University of California, Los Angeles, Working Paper.

Kaplan, A., 1998, "Essays on Financing Decisions: Redemption Features and the Joint Capital Structure/Debt Maturity Decisions," Unpublished PhD Thesis, York University.

Kim, J., K. Ramaswamy, and S. Sundaresan, 1993. "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds? A Contingent Claims Model," *Financial Management* 22, 117-131.

King, T.D. and D.C. Mauer, 2000, "Corporate Call Policy for Nonconvertible Bonds," *Journal of Business* 73, 403-444.

Kwan, S.H. and W.T. Carleton, 2004, "Financial Contracting and the Choice between Private Placements and Publicly Offered Bonds," Federal Reserve Bank of San Francisco Working Paper No. 2004-20.

Litterman, R. and T. Iben, 1991, "Corporate Bond Valuation and the Term Structure of Credit Spreads," *Journal of Portfolio Management*, Spring 52-64.

Livingston, M. and R.E. Miller, 2000, "Investment Bank Reputation and the Underwriting of Nonconvertible Debt," *Financial Management* 29, 21-34.

Livingston, M. and L. Zhou, 2002, "The Impact of Rule 144a Debt Offerings upon Bond Yields and Underwriter Fees," *Financial Management* 31, 5-27.

Longstaff, F.A. and B.A. Tuckman, 1994, "Call Nonconvertible Debt and the Problem of Related Wealth Transfer Effects," *Financial Management* 23, 21-27.

Lyden, S. and D. Saraniti, 2000, "An Examination of the Classical Theory of Corporate Security Valuation," Bourne Lyden Capital Partners Working Paper.

Maddala, G.S., 1983, *Limited-Dependent and Qualitative Variables in Econometrics*, Cambridge, Cambridge University Press.

Mann, S.V. and E.A. Powers, 2003a, "What Is the Cost of a Make-Whole Call Provision?" *Journal of Bond Trading and Management* 1, 315-325.

Mann, S.V. and E.A. Powers, 2003b, "Indexing a Bond's Call Price: An Analysis of Make-Whole Call Provisions," *Journal of Corporate Finance* 9, 535-554.

Mann, S.V. and E.A. Powers, 2007, "Determinants of Bond Tender Offer Premiums and Tendering Rates," *Journal of Banking and Finance* 31, 547-566.

Mauer, D.C., 1993, "Optimal Bond Call Policies under Transactions Costs," *Journal of Financial Research* 16, 23-27.

Merton, R.C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance* 29, 449-470.

Narayanan, M.P. and S.P. Lim, 1989, "On the Call Provision in Corporate Zero-Coupon Bonds," *Journal of Financial and Quantitative Analysis* 24, 91-103.

Powers, E.A. and S. Sarkar, 2006, "Setting Optimal Make-Whole Call Premiums," University of South Carolina Working Paper.

Schultz, P., 2001, "Corporate Bond Trading Costs: A Peek behind the Curtain," *Journal of Finance* 56, 677-698.

Smith, C. and J. Warner, 1979, "On Financial Contracting: An Analysis of Bond Covenants," *Journal of Financial Economics* 7, 117-161.

Wei, D.G. and D. Guo, 1997, "Pricing Risky Debt, an Empirical Comparison of the Longstaff, Schwartz, and Merton Models," *Journal of Fixed Income* September 8-28.